Chapter 11: Quadratics

Quadratics are mathematical expressions containing a term to the second degree with a standard form of

$$\boldsymbol{y = ax^2 + bx + c}$$

For quadratics on the SAT, you need to be able to multiply binomials (FOIL), factor, interpret quadratics on a graph, and use the quadratic equation.

Multiplying Binomials

In order to multiply binomials, you will need to FOIL, which stands for First, Outer, Inner, Last. You are likely familiar with how to multiply binomials by now, but for a quick review, here is an example of how to FOIL:

To multiply $(2x + 3)(x + 6)$ ...

First terms: $(2x)(x) = 2x^2$

Outer terms: $(2x)(6) = 12x$

Inner terms: $(3)(x) = 3x$

Last terms: $(3)(6) = 18$

so we get: $(2x + 3)(x + 6) = 2x^2 + 12x + 3x + 18 = \mathbf{2x^2 + 15x + 18}$

Multiplying Perfect Squares – Don't Forget to FOIL

When multiplying perfect squares, make sure to avoid the common mistake of forgetting to FOIL.

$$(x + 5)^2 \neq x^2 + 25$$

To help avoid this mistake, you can write the perfect square as two terms and then FOIL.

$$(x + 5)^2 = (x + 5)(x + 5) = x^2 + 5x + 5x + 25$$

Combine like terms to get

$$(x + 5)^2 = x^2 + 10x + 25$$

Example 1: Which of the following is equivalent to $(2x + 3)^2 + 4x$?

A) $4x^2 + 16x + 9$ B) $4x^2 + 4x + 9$ C) $2x^2 + 4x + 3$ D) $6x^2 + 9$

Solution: To solve, multiply out the squared term and then combine like terms.

$$(2x + 3)^2 + 4x$$

$$(2x + 3)(2x + 3) + 4x$$

$$4x^2 + 12x + 9 + 4x$$

$$4x^2 + 16x + 9$$

The answer is A.

Don't forget the shortcut! In Chapter 3, we discussed an easy way to solve any "Equivalent to" questions: plug in $x = 1$ and solve. Solving questions similar to Example 1 algebraically works but plugging in $x = 1$ can be faster and easier for more challenging questions.

Factoring Quadratics

You also need to know how to factor quadratics. Factoring can help you simplify expressions or identify the solution(s) to a quadratic equation.

The "Box" Method

We can use the "box" method to find the factors for a quadratic equation. The factors appear on the outside of the box and the quadratic appears in the box.

To see how this works, let's factor the quadratic below:

$$\boldsymbol{f(x) = 3x^2 - x - 2}$$

1. Place the x^2 term in the top left of the box. Place the number in the bottom right.
2. Write down the two terms that must multiply to the top left term outside the box. In this example, $3x$ and x multiply to $3x^2$.

	x	
$3x$	$3x^2$	
		-2

3. Identify which number(s) can multiply to the number in the bottom right of the box. In this example, we need a pair of numbers that multiply to -2. The possibilities are 1 and -2, 2 and -1.
4. Place the pairs of terms outside the box. You have the correct setup when the two other boxes (the x-terms) add up to the middle term in the quadratic. In this example, $-3x + 2x = -x$, so we know the numbers are correctly set up. The factors appear on the outside of the box.

	x	-1
$3x$	$3x^2$	$-3x$
2	$2x$	-2

5. Write down the quadratic in factored form. $\boldsymbol{3x^2 - x - 2 = (3x + 2)(x - 1)}$

There are other ways to factor quadratics. If you know a different method that works for you, use that method. We will not review factoring beyond this example in this book. If you need to review factoring, look up some lessons and practice problems online and in your textbooks.

Solutions, Roots, x-intercepts, and Zeros for Quadratic Equations

The SAT may ask you to find the "solutions," "roots," x-intercepts," or "zeros" of a quadratic equation. All of these terms refer to the values of x that make $f(x) = 0$. Remember, all of these terms mean the same thing. We will refer to these terms collectively as the "solutions" in the rest of this chapter.

To find the solutions, set the quadratic equation equal to zero and factor. Let's continue with the example we are currently working on.

$$3x^2 - x - 2 = 0$$

We just showed how we can factor this quadratic to get

$$(3x + 2)(x - 1) = 0$$

To find the solutions, set each factor equal to zero and solve.

$$3x + 2 = 0 \qquad\qquad x - 1 = 0$$

$$x = -\frac{2}{3} \qquad \text{and} \qquad x = 1$$

The solutions are $x = -\frac{2}{3}$ and $x = 1$.

The solutions are also the x-intercepts if the quadratic is graphed (more on this on the next page).

Example 2: What is the sum of the solutions of the polynomial $f(x) = x^2 - 11x + 18$?

Solution: To solve, we need to find the values of x that make $f(x) = 0$.

$$x^2 - 11x + 18 = 0$$

$$(x-2)(x-9) = 0$$

$$x = 2, 9$$

The solutions are 2 and 9. The sum of the roots is $2 + 9 = 11$. **The answer is 11.**

Shortcut Method: For any sum of solutions question, we can use the rule below:

For any quadratic equation where $ax^2 + bx + c = 0$, the sum of the solutions to a quadratic is always equal to $-\frac{b}{a}$.

For the equation in Example 2, $a = 1$ and $b = -11$, so

$$-\frac{b}{a} = -\frac{-11}{1} = 11$$

The answer is 11. Make sure to memorize this rule, as it is very useful for any questions that asks for the sum of solutions to a quadratic equation.

Example 3: If (x, y) is a solution to the system of equations below, what is a possible value of y given that $x > 0$?

$$y = 2x - 3$$
$$y = x^2 + 12x - 27$$

A) 1 B) 2 C) 8 D) 12

Solution: This system of equations question involves quadratics. We will more thoroughly cover how to solve systems of equations in Chapter 12. When solving a system of equations, the solution(s) are the intersection point(s) of the two functions. Here, we are given the equations

$$y = 2x - 3$$
$$y = x^2 + 12x - 27$$

The easiest way to solve this system of equations is to set the equations equal to each other and solve for x.

$$x^2 + 12x - 27 = 2x - 3$$

$$x^2 + 10x - 24 = 0$$

$$(x-2)(x+12) = 0$$

$$x = 2, -12$$

Since the question specifies that $x > 0$, we must use $x = 2$. To find the y, plug in $x = 2$ to either of the original equations. Here, we will use the easier first equation.

$$y = 2(2) - 3$$

$$y = 1$$

Therefore, a point of intersection is at $(2, 1)$. **The answer is A**.

Example 4: Tom's math teacher is offering to buy Tom's sandwich. The teacher writes the equation $x^2 - 11x + 14 = 26$ on the board and says he will pay Tom *t* dollars for the sandwich, where *t* is equal to the positive solution to the equation on the board. What is the value of *t?*

Solution:

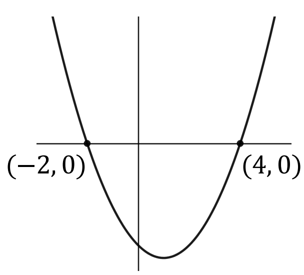
$$x^2 - 11x + 14 = 26$$
$$x^2 - 11 - 12 = 0$$
$$(x - 12)(x + 1) = 0$$
$$x = 12, -1$$

The question tells us that t is positive, so **the answer is 12.** Notice how we had to subtract the 26 before factoring. **You cannot factor a quadratic until the equation is set equal to 0.** This is a very common mistake that students make, so make sure you remember this critical step.

How Solutions Appear on a Graph

Solutions appear as the x-intercepts when graphed in the xy-plane. When we have a quadratic or other polynomial in factored form, we can see where the x-intercepts are. We will review the rules for multiplicity (the power to which a factor is raised) and zeros for polynomial functions below:

$$y = (x + 2)(x - 4)$$

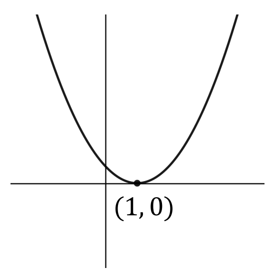


Multiplicity = 1

Zeros: The function has 2 solutions at $x = -2$ and $x = 4$.

Behavior: The function passes straight through the x-axis at the solution.

$$y = (x - 1)^2$$

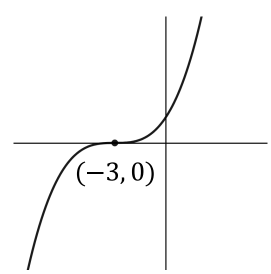


Multiplicity = 2

Zeros: The function has 1 solution at $x = 1$.

Behavior: The function bounces at the solution and does not cross the x-axis.

$$y = (x + 3)^3$$



Multiplicity = 3

Zeros: The function has 1 solution at $x = -3$.

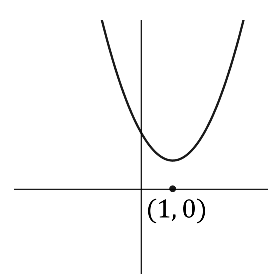
Behavior: The function flattens and passes through the x-axis at the solution.

TIP – Functions with No Real Solution

If a function never crosses the x-axis, the function has no real solution. In other words, the function has no x-intercept.

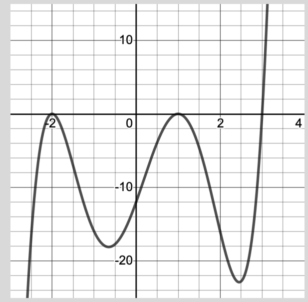
As an example, the function $f(x)$ to the right has no real solution. This function cannot be factored to solve for x. If you use the quadratic formula too solve, the solutions are imaginary numbers.

$$f(x) = x^2 - 2x + 2$$



Example 5: Which of the following equations correctly describes the function in the graph below?

A) $y = (x+2)^2(x-1)^2(x-3)$
B) $y = (x+2)(x-1)(x-3)$
C) $y = (x-2)^2(x+1)(x+3)$
D) $y = (x+2)(x-1)^2)(x-3)^2$



Solution: To solve this question, we need to look at the behavior of the function at each of the x-intercepts. There are x-intercepts at $x = -2$, $x = 1$, and $x = 3$, so we need to see the factors $(x+2)$, $(x-1)$, and $(x-3)$ in the correct answer. Now, we need to find out what power each term should be raised to. At $x = -2$ and $x = 1$, the function bounces, so the $(x+2)$ and $(x-1)$ terms are squared. At $x = 3$, the function goes straight through, so the $(x-3)$ should be to the first power. **The answer is A.**

The Quadratic Formula

If a quadratic is not easily factorable, you will need to use the quadratic formula to solve for the roots of a quadratic function. You will need to have the quadratic formula memorized.

For $\boldsymbol{ax^2 + bx + c = 0}$, the solution(s) are given by: $\boldsymbol{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Example 6: Which of the following is a solution for the function $f(x) = x^2 - 8x + 4$?

A) $8 + 4\sqrt{2}$ B) $4 - 2\sqrt{3}$ C) $-4 + 2\sqrt{3}$ D) $-8 + 4\sqrt{3}$

Solution: Since this quadratic cannot be factored, we must use the quadratic formula.

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(4)}}{2(1)}$	1. Plug in the values for a, b, and c.
$x = \frac{8 \pm \sqrt{64-16}}{2}$	2. Begin to simplify terms.
$x = \frac{8 \pm \sqrt{48}}{2}$	3. Combine terms under the radical.
$x = \frac{8 \pm 4\sqrt{3}}{2}$	4. Simplify radical (if possible).
$x = 4 \pm 2\sqrt{3}$	5. Simplify terms further (if possible).
$x = 4 + 2\sqrt{3}$ and $x = 4 - 2\sqrt{3}$	6. Identify the value(s) of x.

The answer is B.

Example 7: The equations below intersect at the point (x, y). Which of the following is a value of x?

$$y = x^2 + 3x + 15$$

$$y = -3x + 11$$

A) 2 B) $6 + \sqrt{5}$ C) $-3 + \sqrt{13}$ D) $-3 - \sqrt{5}$

Solution: This is another system of equations question like the one we solved in Example 3. The solutions to the system of equations are where the functions intersect. The easiest way to solve this system of equations is to set the equations equal to each other and solve for x.

$$x^2 + 3x + 15 = -3x + 11$$

$$x^2 + 6x + 4 = 0$$

At this point, we cannot easily factor, so we need to use the quadratic formula.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = -3 \pm \sqrt{5}$$

The x-values at the points of intersection for the system are at $x = -3 + \sqrt{5}$ and $x = -3 - \sqrt{5}$. **The answer is D.**

The Discriminant

In the quadratic formula, the discriminant is the $b^2 - 4ac$ term under the radical. This term is very important because it can quickly tell us how many real or complex solutions there will be for any quadratic equation. **The exact value of the discriminant is not important, but whether it is positive, negative, or zero is.**

Discriminant Value	Types of Solutions
$\boldsymbol{b^2 - 4ac > 0}$	**2 real solutions**
$\boldsymbol{b^2 - 4ac = 0}$	**1 real solution**
$\boldsymbol{b^2 - 4ac < 0}$	**0 real solutions, 2 complex solutions**

Memorize these rules. **If you ever see a question about the number of solutions to a system of equations, use the discriminant to solve.**

Example 8: How many real solutions are there to the function below?

$$h(x) = 2x^2 - 7x + 9$$

A) 0 B) 1 C) 2 D) 3

Solution:

$$\text{Discriminant} = b^2 - 4ac = (-7)^2 - 4(2)(9)$$

$$\text{Discriminant} = -23$$

The discriminant is negative, so there are no real solutions. **The answer is A.**

Example 9: In the system of equations below, m is a constant. For which of the following values of m does the system of equations have exactly 1 real solution?

$$y = x^2 - 8x + 10$$

$$y = m - 2x$$

A) -1 B) 0 C) 1 D) 2

Solution: To start, we set the equations equal.

$$x^2 - 8x + 10 = m - 2x$$

$$x^2 - 6x + (10 - m) = 0$$

If the system of equations has one real solution, the equation above must have one real solution and the discriminant must be equal to 0.

$$\text{Discriminant} = b^2 - 4ac = (-6)^2 - 4(1)(10 - m) = 0$$

At this point, you can either test each of the answer choices to see which one makes the discriminant equal 0 or solve algebraically for m. The steps below show how to solve algebraically.

$$36 - 4(10 - m) = 0$$

$$36 - 40 + 4m = 0$$

$$-4 + 4m = 0$$

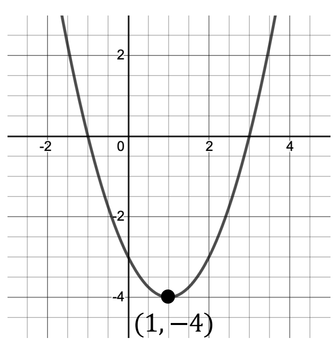
$$4m = 4$$

$$m = 1$$

The answer is C.

The Vertex

The vertex is the maximum or minimum of a parabola. For the parabola shown below, the vertex is at $(1, -4)$.



You can think of the vertex as the midpoint of a parabola because the x-coordinate of the vertex is always the midpoint of the two solutions, or x-intercepts. More simply, the x-coordinate of the vertex is equal to the average of the solutions. Using the parabola above as an example, we can see how this works. Since the solutions (x-intercepts) for the parabola are located at $x = -1$ and $x = 3$, the x-coordinate of the vertex is at $x = \frac{-1+3}{2} = 1$, which matches the graph.

Remember, the vertex is always the maximum or minimum value of a quadratic. For the example above, the minimum is at $y = -4$.

Example 10: For the equation $f(x) = (x-6)(x+2)$, what is the value of x at the minimum value of the function?

Solution: The minimum value of a parabola is at the vertex, so we need to find the x-coordinate of the vertex. Since the function is already in factored form, we see the roots are at $x = 6$ and $x = -2$. The x-coordinate of the vertex is the average of the roots, so we can find

$$x = \frac{6+(-2)}{2} = 2$$

The x-coordinate of the vertex is at $x = 2$, so **the answer is 2.**

The average method works perfectly if you are given a quadratic that is already factored. But what if you are not given a quadratic in factored form? Good news! There is a second way to quickly find the vertex.

For any quadratic in the form of $ax^2 + bx + c$, you can find the x-coordinate of the vertex using

$$\boldsymbol{x = -\frac{b}{2a}}$$

Make sure you memorize this equation! It can really help you quickly and easily solve any questions where you need to find the vertex of a parabola.

Example 11: Andre runs a business that sells used cameras. To prepare the cameras for sale, Andre must spend x hours repairing and cleaning each camera. The equation $P(x) = -0.05x^2 + 0.4x + 0.9$ models the percentage of profit, *P*, that Andre gets from selling each camera he spent x hours repairing and cleaning. Which of the following shows the number of repair hours that maximizes Andre's percentage of profit for selling each camera?

A) 3 B) 4 C) 6 D) 8

Solution: To determine the number of repair hours that maximize Andre's percentage of profit, we need to find the vertex of the equation. Since the leading value in the parabola is negative, the parabola is downward facing, so the vertex will be the maximum of the graph. The number of hours will be on the x-axis, so we need to find the x-coordinate of the vertex to solve.

To find the x-coordinate of the vertex, you can use the equation we just introduced above:

$$x = -\frac{b}{2a} = -\frac{0.4}{2(-0.05)} = 4$$

The answer is B.

Example 12: Claire's Big Top Circus has two high-flying acts: a man shot out of a cannon and a woman on a zip line. Claire wants to see if she can run the two high-flying acts at the same time without risking collision. The position of the man shot out of a cannon can be modeled by the equation $C(x) = -x^2 + 30x + 200$, where x represent seconds after launch. The zip line follows the equation $Z(x) = -5x + 150$, where x represents seconds after takeoff. Which of the following is the best advice to give Claire?

A) The two high-flying acts cannot run at the same time because the performers will collide.
B) The two high-flying acts can run at the same time because the performers will not collide.
C) The two high-flying acts will be risky because the performers might sometimes collide.
D) More information is needed to determine whether the high-flying acts can run at the same time.

Solution: We can solve this problem mathematically or by using a graphing calculator.

Method #1 – "Math Teacher Way": We are asked to determine whether the man shot out of a cannon and the woman will collide. In math terms, we are asked to determine if the equations $C(t)$ and $Z(t)$ will intersect and, if they do intersect, where the intersection will occur. Since we are given two equation, we need to solve the system of equations. We are given

$$C(t) = -x^2 + 30x + 200$$

$$Z(t) = -5x + 150$$

To solve, we can set the equations equal to each other.

$$-x^2 + 30x + 200 = -5x + 150$$

$$-x^2 + 35x + 50 = 0$$

At this point, we cannot easily factor, so we need to use the quadratic equation.

$$x = \frac{-35 \pm \sqrt{35^2 - 4(-1)(50)}}{2(-1)}$$

$$x = \frac{-35 \pm \sqrt{1425}}{-2}$$

$$x = \frac{-35 \pm 5\sqrt{57}}{-2}$$

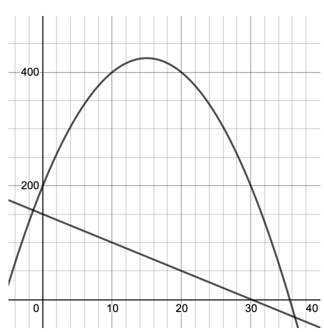
$$x = -1.37, 36.37$$

We see the system will have solutions at $x = -1.37$ and $x = 36.37$. The solution at $x = -1.37$ will not be a point where the man and woman will collide because it is negative, and we cannot have a negative value for time. Now, we need to determine whether the intersection at $x = 36.37$ will or will not be a collision point. To do so, we can plug $x = 36.37$ into either of the initial equations.

$$Z(t) = -5(36.37) + 150 = -31.85$$

The functions will intersect at the point $(36.37, -31.85)$. Since y-value at this point is negative and the y-value represents the height, this will also not be a point of collision. By the point when $x = 36.37$, the man and woman will have already landed safely. As a result, the two high-flying acts can run at the same time without colliding. **The answer is B.**

Method #2 – Graphing the Equations: If you have a graphing calculator, we can skip this math and simply graph the equations. You can see on the graph below that the man and woman will not collide during the performance. You can also see the two points of intersection at $x = -1.37$ and $x = 36.37$ that we solved for above. **The answer is B.**



That is a very difficult question! Of course, graphing this equation is the easiest way to solve. If you have a graphing calculator, use it! But make sure you understand how we solved this mathematically in case a question similar to this come up on the no-calculator section (it would, of course, have easier numbers).

<u>Quadratics Practice</u>: A calculator may NOT be used on the following questions. Answers on page 244.

1. What is the sum of the 4 binomials listed below?

$$x^2+3, 4x+6,\ 3x^2+1, 3x-1$$

A) $4x^2+7x+9$
B) $4x^2+7x+11$
C) $7x^2+4x+9$
D) $4x^2+4x+11$

2. Which of the following is equivalent to $(3x-5)(-x+7)$?

A. $(3x+5)(x+7)$
B. $(3x-5)(x+7)$
C. $(-3x+5)(x-7)$
D. $(-3x+5)(x+7)$

3. What is the sum of the solutions of the polynomial $f(x)=x^2-7x+12$?

A) -7
B) 3
C) 4
D) 7

4. What are the solutions of the quadratic equation $3x^2+9x-12=0$?

A) $x=1$ and $x=4$
B) $x=-1$ and $x=4$
C) $x=-1$ and $x=-4$
D) $x=1$ and $x=-4$

5. If (x,y) is a solution to the system of equations below, what is a possible value of x?

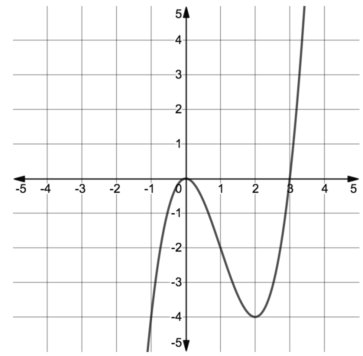
$$y=x^2+6x+6$$
$$y=2x+2$$

A) -2
B) 0
C) 2
D) 4

6. What is the sum of the solutions of the equation $x^2-4x-21=0$?

A) -10
B) -4
C) 3
D) 4

7. The function $f(x)$ is graphed below. Which of the following could define the function $f(x)$?



A. $f(x)=x(x-3)$
B. $f(x)=x(x+3)$
C. $f(x)=x^2(x-3)$
D. $f(x)=x^2(x+3)$

8. In the xy-plane, the parabola with equation $y=(x+3)(x+4)$ intersects the equation $y=20$ at two points. Which of the following is an x-value of a point of intersection?

A) -8
B) -1
C) 3
D) 4

9. In the equation below, a and b are constants. Which of the following could be the value of a?

$$9x^2-16=(ax-b)(ax+b)$$

A) 3
B) 4
C) 9
D) 16

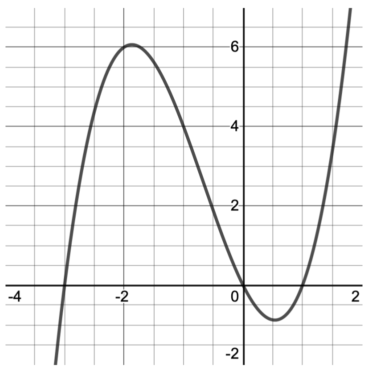
10. Which of the following is equivalent to the expression below?

$$x^2+8x+8$$

A) $(x+4)^2-8$
B) $(x+4)^2+8$
C) $(x-4)^2-8$
D) $(x-4)^2+8$

11. What is the sum of the solutions to $(x-1.2)(x+5)=0$?

A) -6.2
B) -3.8
C) 3.8
D) 6.2

12. 

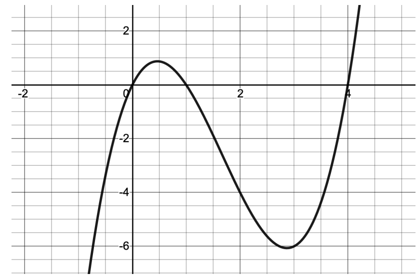
The function $f(x)$ is graphed above. Which of the following could define the function $f(x)$?

A) $f(x)=(x-3)(x+1)$
B) $f(x)=x(x-3)(x+1)$
C) $f(x)=(x+3)(x-1)$
D) $f(x)=x(x+3)(x-1)$

13. In the equation below, j, l, k, and m are constants. If the equation has roots of $-4, 3, \text{and} -5$. Which of the following could be a factor of the equation below?

$$jx^3+lx^2-kx-m=0$$

A) $x-4$
B) $x-5$
C) $x-3$
D) $x+3$

14. 

Which of the following correctly models the graph above?

A) $(x-1)(x-4)$
B) $x(x-1)(x-4)$
C) $(x+1)(x+4)$
D) $x(x+1)(x+4)$

15. Which of the following is a solution to the equation below?

$$x^2+6x+3=0$$

A) $-3+\sqrt{6}$
B) $-3+\sqrt{13}$
C) $3-\sqrt{6}$
D) $3-\sqrt{13}$

16. What is the sum of the solutions to the given equation?

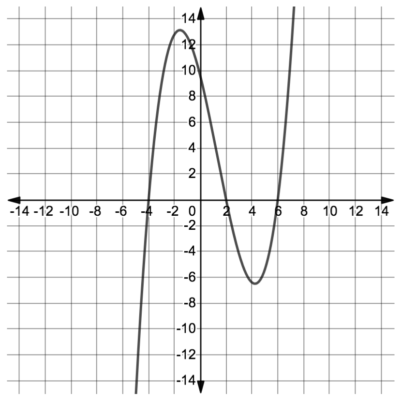
$$x^2-13x+40=6x-8$$

A) -13
B) -19
C) 13
D) 19

17. The system of equations below is graphed in the xy-plane. Which of the following is the x-coordinate of an intersection point (x, y) of the system of equations?

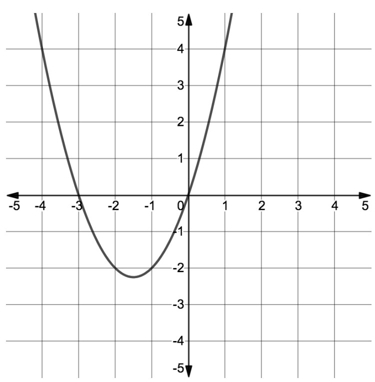
$$y=x^2+8x+9$$
$$y=2x+3$$

A) $-3+\sqrt{3}$
B) $3+2\sqrt{3}$
C) $-5-\sqrt{13}$
D) $3-\sqrt{3}$

18. 

The function $g(x)$ is graphed above. Which of the following could define the function $g(x)$?

A) $g(x) = \frac{1}{5}(x-4)(x+2)(x+6)$
B) $g(x) = -\frac{1}{5}(x-4)(x+2)(x+6)$
C) $g(x) = \frac{1}{5}(x+4)(x-2)(x-6)$
D) $g(x) = -\frac{1}{5}(x+4)(x-2)(x-6)$

19. 

The function $f(x)$ is graphed above. Which of the following could define the function $f(x)$?

A) $f(x) = x^2 + 3x$
B) $f(x) = x^2 - 3x$
C) $f(x) = -x^2 + 3x$
D) $f(x) = -x^2 - 3x$

20. Given that $(2x+3)$ and $(x-4)$ are the factors of the quadratic below, what is the value of z?

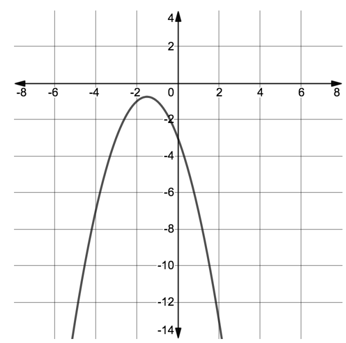
$$2x^2 + (z-1)x + 2z - 4$$

A) -2
B) -4
C) 2
D) 8

21. $$x^3 + 8x^2 - 27x - 28 = 0$$

The polynomial above can be written as $(x+1)(x+7)(x^2-4) = 0$. What are all of the roots of the equation?

A) $-1, -7$
B) $-1, -7, \sqrt{2}$
C) $-2, -1, 2, 7$
D) $-7, -2, -1, 2$

22. 

The function $f(x)$ is graphed above. Which of the following could define the function $f(x)$?

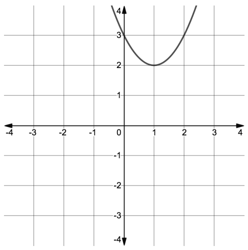
A) $f(x) = x^2 - 2x - 3$
B) $f(x) = x^2 - 3x - 3$
C) $f(x) = -x^2 - 3$
D) $f(x) = -x^2 - 3x - 3$

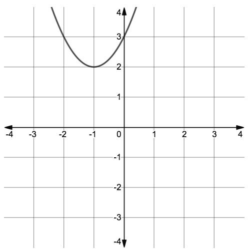
23. The system of equations below is graphed in the xy-plane. Which of the following is the sum of the values of the two x-coordinates of the intersection points (x, y)?

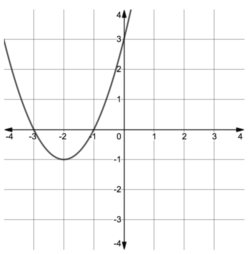
$$y = x^2 + 2x + 1$$
$$y = -3x - 3$$

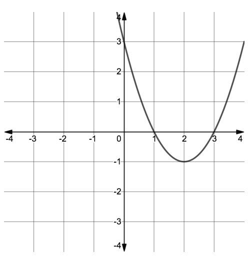
A) -5
B) -3
C) 3
D) 5

24. Which of the following could be the graph of $f(x) = x^2 - 2x + 3$?

A) 

B) 

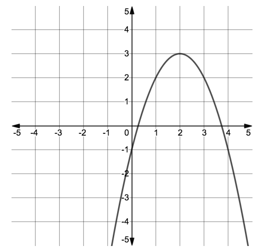
C) 

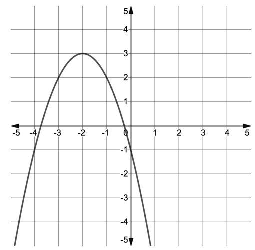
D) 

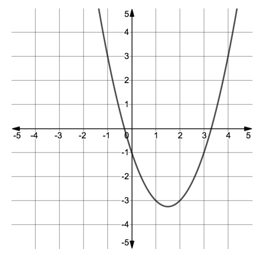
25. The system of equations below is graphed in the xy-plane. If x is not a negative number, what is a possible value of x?

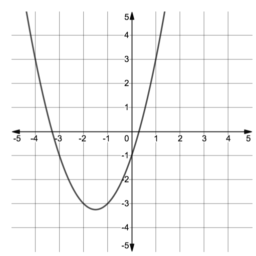
$$y = x^2 + 5x + 8$$
$$y = 8 - 2x$$

26. Which of the following could be the graph of $f(x) = -x^2 + 4x - 1$?

A) 

B) 

C) 

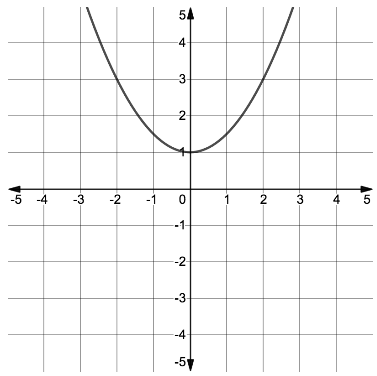
D) 

27. Which of the following is a solution to the equation below?

$$x^2 - 4x + 1 = 0$$

A) $2 - \sqrt{6}$
B) $-2 + \sqrt{6}$
C) $2 + \sqrt{3}$
D) $-2 - \sqrt{3}$

28. The function $f(x)$ is graphed above. Which of the following could define the function $f(x)$?



A) $f(x) = \frac{1}{2}(x-2)(x+2)$
B) $f(x) = (x-2)(x+2)$
C) $f(x) = \frac{1}{2}(x-2)(x+2)+1$
D) $f(x) = \frac{1}{2}(x-2)(x+2)+3$

29. In the quadratic equation below, z is a constant. For what value of z, will the equation have one real solution?

$$zx^2 + 6x = 3$$

A) -3
B) 1
C) 3
D) 6

30. The equation below is graphed in the xy-plane. If a and b are positive constants and $a \neq b$, how many distinct x-intercepts does the graph have?

$$x^2 + ax + bx + ab = 0$$

A) 0
B) 1
C) 2
D) 3

<u>Quadratics Practice</u>: A calculator may be used on the following questions.

31. What is the solution set for $5x^2 + 6x = 8$?

A) $\{\frac{1}{5}, \frac{1}{2}\}$
B) $\{-\frac{1}{5}, -\frac{1}{2}\}$
C) $\{\frac{4}{5}, 2\}$
D) $\{\frac{4}{5}, -2\}$

32. If (x, y) is a solution to the system of equations below, what is a possible value of $y - x$?

$$y = x^2 + 9x + 8$$
$$y = 11x + 7$$

A) -1
B) 1
C) 17
D) 18

33. What is the sum of the solutions to the given equation?

$$x^2 - 12x + 26 = 2x + 2$$

A) 14
B) 11
C) -11
D) -14

34. How many solutions $(x. y)$ are there to the system of equations below?

$$y = x^2 + 11x + 4$$
$$y = 5x - 5$$

A) 0
B) 1
C) 2
D) 4

35. $(80x - 42)(15x + 12) = ax^2 + bx + c$

For the equation above, what is the value of $a + b + c$?

36. $(kx + 3)(4x^2 - mx - 3) = 20x^3 - 3x^2 - 24x - 9$

For the equation above, what is the value of km?

A) -15
B) -5
C) 3
D) 15

37. $ax^3 + bx^2 + cx + d = 0$

In the function above, a, b, c, and d are all constants. If the equation has roots at -3, 6, and 8, which of the following is a factor of $ax^3 + bx^2 + cx + d$?

A) $x + 1$
B) $x + 3$
C) $x - 3$
D) $x + 6$

38. $h(x) = x^4 + 2x^3 - 8x^2 - 18x - 9$

The polynomial above can be written as $(x^2 - 9)(x + 1)^2$. What are all the real roots of the equation?

A) $9, 1$
B) $9, 1, \text{and} -1$
C) $3, -3, \text{and} -1$
D) $3, -3, 1, \text{and} -1$

39. The equation below is graphed in the xy-plane. If a and b are positive constants and $a \neq b$, how many distinct x-intercepts does the graph have?

$$y = (x + a)(x - a)(x + b)^2$$

A) 1
B) 2
C) 3
D) 4

40. Ben is throwing a ball from the top of his building. The ball's height is modeled by the function $H(x) = -x^2 + 10x + 56$, where x is the number of seconds after he throws the ball. How many seconds after throwing the ball does it hit the ground?

A) 4
B) 5
C) 14
D) 56

41. Which of the following is a solution to the equation below?

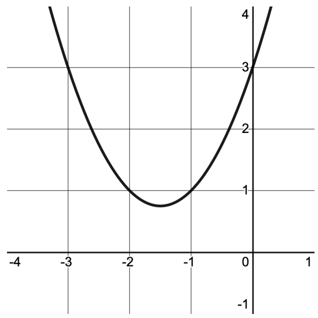
$$2x^2 + 4x + 1 = 0$$

A) $\frac{-2+\sqrt{2}}{2}$
B) $2\sqrt{2}$
C) $-2 + \sqrt{2}$
D) $\frac{2-\sqrt{2}}{2}$

42. John is launching a rocket. The rocket's height in feet is modeled by the function $H(x) = -x^2 + 30x$, where x is the number of seconds after launch. What is the maximum height of the rocket?

A) 10 feet
B) 15 feet
C) 200 feet
D) 225 feet

43.



$$f(x) = x^2 + bx + 3$$

The graph and equation of the function $f(x)$ are shown above. Which of the following is the value of b?

A) 2
B) 3
C) 5
D) 8

44. Which of the following functions has a graph in the xy-plane with no x-intercepts?

A) $y = 6(x-3)^3$
B) $y = 2x + 9$
C) $y = x^2 + 8x + 7$
D) $y = x^2 + 4x + 6$

45. Which of the following is a solution to the equation below?

$$5x^2 + 4x - 4 = 6x - 2$$

A) $\frac{4}{5}$
B) $\frac{1-\sqrt{11}}{5}$
C) $\frac{1}{5} - \sqrt{11}$
D) $\frac{\sqrt{11}}{5}$

46. Dave and Charlie are trying to perform a trick shot. Dave will be on a zip line and his height is represented by the function $f(x) = -3x + 50$, where x represents seconds after starting the zipline. Charlie will be throwing a baseball and the baseball's height is represented by the function $g(x) = -x^2 + 6x + 30$, where x represents the number of seconds after Charlie releases the ball. In order to perform the trick shot, Dave and the baseball must intersect. If Dave starts the zipline at the same time as Charlie releases the ball, which of the following is the most appropriate conclusion about if the trick shot is possible?

A) The trick shot is not possible because the two functions do not intersect.
B) The trick shot is possible and there is only one point of intersection.
C) The trick shot is possible and there are two points of intersection.
D) More information is needed to figure out whether the trick shot is possible.

47.

$$y = x - 18$$
$$y^2 + (x-14)^2 - 12 = 0$$

For the system of equations above, what is a possible value of y?

A) $-2 + \sqrt{2}$
B) $\sqrt{2}$
C) $2 + \sqrt{3}$
D) $-2 - \sqrt{6}$

48. In the system of equations below, a and b are constants. For which of the following values of a and b does the system of equations have exactly one real solution?

$$y = 6x + 2$$
$$y = ax^2 + b$$

A) $a = 3,\ b = 1$
B) $a = 3,\ b = 3$
C) $a = 9,\ b = 3$
D) $a = -3,\ b = 1$

Answers and Answer Explanations

Chapter 11 – Quadratics (Pages 62 – 68)

1. **A** Combine like terms. $x^2 + 3x^2 = 4x^2$. $4x + 3x = 7x$. $3 + 6 + 1 - 1 = 9 \rightarrow 4x^2 + 7x + 9$
2. **C** Step 1: FOIL and combine like terms. $(3x - 5)(-x + 7) = -3x^2 + 26x - 35$
   Step 2: Check each answer choice to see which is the same.
   Alternative method: Plug in $x = 1$ to the initial equation and the answer choices and see which one is the same.
   Original equation: $(3(1) - 5)(-1 + 7) = -12$. Correct answer: $(-3(1) + 5)(1 - 7) = -12$
3. **D** Method 1: Step 1: Factor and solve for the solutions. $x^2 - 7x + 12 = 0 \rightarrow (x - 4)(x - 3) = 0 \rightarrow$ $x = 4, 3$
   Step 2: Add the solutions. $3 + 4 = 7$
   Method 2: Use $-\frac{b}{a}$ to find sum of the solutions. $-\frac{-7}{1} = 7$
4. **D** Factor and solve for solutions. $3x^2 + 9x - 12 = 0 \rightarrow (3x - 3)(x + 4) = 0 \rightarrow 3x - 3 = 0$ and $x + 4 = 0 \rightarrow x = 1$ and $x = -4$
5. **A** Step 1: Set equal. $x^2 + 6x + 6 = 2x + 2$
   Step 2: Move all of the values to one side to set equal to 0 and factor. $x^2 + 4x + 4 = 0 \rightarrow$ $(x + 2)(x + 2) = 0 \rightarrow x = -2$
6. **D** Method 1: Step 1: Factor and solve for solutions. $x^2 - 4x - 21 = (x - 7)(x + 3) \rightarrow x = 7, -3$
   Step 2: Add the solutions. $7 + (-3) = 4$
   Method 2: Use $-\frac{b}{a}$ to find sum of the solutions. $-\frac{-4}{1} = 4$
7. **C** Step 1: Identify the solutions or x-intercepts on the graph are at $x = 0, 3$
   Step 2: To identify whether the factors should be $x(x - 3)$ or $x^2(x - 3)$, look at the behavior of the graph at the intercept. Since the graph bounces at $x = 0$ and passes straight through at $x = 3$, the factors must be $x^2(x - 3)$.
8. **A** Step 1: Set equal. $20 = (x + 3)(x + 4)$
   Step 2: FOIL the right side of the equation. $20 = x^2 + 7x + 12$
   Step 3: Move all of the values to one side to set equal to 0 and factor. $0 = x^2 + 7x - 8 \rightarrow$ $0 = (x + 8)(x - 1) \rightarrow x = -8, 1$
9. **A** Step 1: FOIL the right side of the equation. $9x^2 - 16 = (ax - b)(ax + b) \rightarrow 9x^2 - 16 = a^2x^2 - b^2$
   Step 2: Solve for a by using the x-terms. $9x^2 = a^2x^2 \rightarrow 9 = a^2 \rightarrow 3 = a$
10. **A** Step 1: For these types of question, FOIL out the answer choices and then combine like terms.
    $(x + 4)^2 - 8 = x^2 + 8x + 16 - 8 = x^2 + 8x + 8$
    Alternative method: Plug in $x = 1$ to the initial equation and the answer choices and see which one is the same.
    Original equation: $1^2 + 8(1) + 8 = 17$. Correct answer: $(1 + 4)^2 - 8 = 17$
11. **B** Step 1: Solve for the solutions by setting each factor equal to 0. $x - 1.2 = 0$ and $x + 5 = 0 \rightarrow$ $x = 1.2, -5$
    Step 2: Find the sum of the two solutions. $-5 + 1.2 = -3.8$
12. **D** Step 1: Identify the solutions or x-intercepts of the graph are at $x = -3, 0, 1$
    Step 2: Use the zeros to find the appropriate factors. $x(x + 3)(x - 1)$
13. **C** Step 1: To find the root of a quadratic, set the factors equal to zero and solve for x. Here, we want to do the reverse. To find the factors, set x equal to the root and solve by setting equal to zero.
    For $x = -4 \rightarrow x + 4 = 0$
    For $x = 3 \rightarrow x - 3 = 0$
    For $x = -5 \rightarrow x + 5 = 0$
    $x - 3$ is the only factor that is an answer choice.
14. **B** Step 1: Find the three x-intercepts of $x = 0$, $x = 1$, and $x = 4$. The factors are $x(x - 1)(x - 4)$.
    Step 2: Find the multiplicity. Since the function go straight through each x-intercept, each solution has a multiplicity of 1.
15. **A** Use the quadratic formula and simplify the square root. $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$

16. **D** Method 1: Step 1: Set equation equal to zero and solve for the solutions. $x^2 - 13x + 40 = 6x - 8 \rightarrow$ $x^2 - 19x + 48 = 0 \rightarrow (x - 16)(x - 3) = 0 \rightarrow x = 16, 3$
Step 2: Add the solutions. $16 + 3 = 19$
Method 2: Step 1: Move all terms to left side. $x^2 - 13x + 40 = 6x - 8 \rightarrow x^2 - 19x + 48 = 0$
Step 2: Use $-\frac{b}{a}$ to find sum of the solutions. $-\frac{-19}{1} = 19$
17. **A** Step 1: Set equal and then move all terms to one side. $2x + 3 = x^2 + 8x + 9 \rightarrow 0 = x^2 + 6x + 6$
Step 2: Since this is not factorable, use the quadratic formula to solve.
$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)} \rightarrow \frac{-6 \pm \sqrt{12}}{2} \rightarrow \frac{-6 \pm 2\sqrt{3}}{2} \rightarrow -3 \pm \sqrt{3}$
18. **C** Step 1: Identify the solutions or x-intercepts of the graph are at $x = -4, x = 2$, and $x = 6$.
Step 2: Use the zeros to find the appropriate factors. $(x + 4)(x - 2)(x - 6)$
Step 3: You don't need to solve for the $\frac{1}{5}$. Based on the shape of the graph, we can tell this will be positive. Since the cubic function starts in the bottom left and finishes in the top right, the coefficient must be positive.
19. **A** Step 1: Identify this is an upward-facing parabola, so the answer must be A or B.
Step 2: $\frac{-b}{2a}$ identifies the x-coordinate of the vertex of the graph. $-\frac{3}{2(1)} = -1.5$
20. **B** Step 1: Set the quadratic and the factors equal and FOIL the left side.
$(2x + 3)(x - 4) = 2x^2 + (z - 1)x + 2z - 4 \rightarrow 2x^2 - 5x - 12 = 2x^2 + (z - 1)x + 2z - 4$
Step 2: The easiest way to solve this question is to plug in the answer choices to see which one makes both sides equal.
Alternatively, solve for z. $2x^2 - 5x - 12 = 2x^2 + (z - 1)x + 2z - 4 \rightarrow -5x - 12 = zx - x + 2z - 4 \rightarrow$ $-4x - 8 = zx + 2z \rightarrow z = -4$
21. **D** Step 1: Factor out $(x^2 - 4)$. $(x^2 - 4) = (x + 2)(x - 2)$ so the equation becomes $(x + 1)(x + 7)(x + 2)(x - 2) = 0$
Step 2: To find the roots, set the factors equal to zero and solve for x. $x + 1 = 0 \rightarrow x = -1$
$x + 7 = 0 \rightarrow x = -7$ $\quad x + 2 = 0 \rightarrow x = -2$ $\quad x - 2 = 0 \rightarrow x = 2$
22. **D** Step 1: Identify this is a downwards facing parabola therefore it must be $-x^2$.
Step 2: $\frac{-b}{2a}$ will identify the x-coordinate of the vertex of the graph. D gives the correct vertex location.
$x = -\frac{-3}{2(-1)} = -\frac{3}{2} = -1.5$
23. **A** Method 1: Step 1: Set equal. $x^2 + 2x + 1 = -3x - 3$
Step 2: Move all of the values to one side to set equal to 0 and factor. $x^2 + 5x + 4 = 0 \rightarrow (x + 4)(x + 1) = 0 \rightarrow x = -4, -1$
Step 3: Add the solutions. $-1 + -4 = -5$
Method 2: Complete the same steps 1 and the first part of 2 to get $x^2 + 5x + 4 = 0$
Step 3: Use $-\frac{b}{a}$ to find sum of the solutions. $-\frac{5}{1} = -5$
24. **A** $\frac{-b}{2a}$ will identify the x-coordinate of the vertex of the graph. $x = -\frac{-2}{2(1)} = 1$
Answer choice A has the only graph with an x-coordinate of the vertex at $x = 1$.
25. **0** Step 1: Set equal. $x^2 + 5x + 8 = 8 - 2x$
Step 2: Move all of the values to one side to set equal to 0 and factor. $x^2 + 7x = 0 \rightarrow x(x + 7) = 0 \rightarrow$ $x = 0 \quad x + 7 = 0 \rightarrow x = 0, -7$. You cannot grid in a negative number on the SAT, so the answer is 0.
26. **A** Step 1: Identify this is a downward-facing parabola, so the answer must be A or B.
Step 2: $\frac{-b}{2a}$ will identify the x-coordinate of the vertex of the graph. $x = -\frac{4}{2(-1)} = 2$
27. **C** Use the quadratic formula. $x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} \rightarrow x = \frac{4 \pm \sqrt{12}}{2} \rightarrow x = \frac{4 \pm 2\sqrt{3}}{2} \rightarrow x = 2 \pm \sqrt{3}$
28. **D** Identify the y-intercept is 1. Plug in $x = 0$ to the answer choices to solve for which one gives the correct y-intercept. $f(0) = \frac{1}{2}(0 - 2)(0 + 2) + 3 \rightarrow f(0) = -2 + 3 \rightarrow f(0) = 1$
29. **A** Step 1: Set the quadratic equal to zero. $zx^2 + 6x - 3 = 0$
Step 2: To find when a quadratic will have one real value, set the discriminant equal to zero.
$b^2 - 4ac = 0 \rightarrow 6^2 - 4(z)(-3) = 0 \rightarrow 36 + 12z = 0 \rightarrow 12z = -36 \rightarrow z = -3$

30. **C** Step 1: Factor out the quadratic. $(x + a)(x + b) = 0$
Step 2: Solve for the factors. $x + a = 0 \rightarrow x = -a \quad x + b = 0 \rightarrow x = -b$
Since a and b are positive constants and do not equal each other, there are two distinct x-intercepts.
Alternative method: Pick different positive values for a and b and solve.
31. **D** Move all of the values to one side to set equal to 0 and factor. $5x^2 + 6x - 8 = 0 \rightarrow$
$(5x - 4)(x + 2) = 0 \rightarrow x = \frac{4}{5}, -2$
32. **C** Step 1: Set equal. $x^2 + 9x + 8 = 11x + 7$
Step 2: Move all of the values to one side to equal 0 and factor. $x^2 - 2x + 1 = 0 \rightarrow (x - 1)^2 = 0 \rightarrow x = 1$
Step 3: Plug $x = 1$ into either equation to solve for y. $y = 11x + 7 \rightarrow y = 11(1) + 7 \rightarrow y = 18$
Step 4: Solve for $y - x$. $18 - 1 = 17$
33. **A** Method 1: Step 1: Move all values to one side and set equal to 0.
Step 2: Factor. $x^2 - 14x + 24 = 0 \rightarrow (x - 12)(x - 2) = 0 \rightarrow x = 2, 12$
Step 2: Add the solutions. $12 + 2 = 14$
Method 2: Use same step 1 as above. Step 2: Use $-\frac{b}{a}$ to find sum of the solutions. $-\frac{-14}{1} = 14$
34. **B** Step 1: Set equal. $x^2 + 11x + 4 = 5x - 5$
Step 2: Move all of the values to one side to set equal to 0 and factor. $x^2 + 6x + 9 = 0 \rightarrow$
$(x + 3)^2 = 0 \rightarrow x = -3$
Alternatively, use the discriminate to find the number of solutions. $b^2 - 4ac = 6^2 - 4(1)(9) = 0$, which means there is 1 solution.
35. **1026** FOIL the two factors on the left side. $(80x - 42)(15x + 12) = ax^2 + bx + c \rightarrow$
$1200x^2 + 330x - 504 = ax^2 + bx + c$ so $a = 1200$, $b = 330$, and $c = -504$
$a + b + c = 1200 + 330 - 504 = 1026$
36. **D** Step 1: FOIL out the left side of the equation. $(kx + 3)(4x^2 - mx - 3) = 20x^3 - 3x^2 - 24x - 9 \rightarrow$
$4kx^3 + 12x^2 - kmx^2 - 3kx - 3mx - 9 = 20x^3 - 3x^2 - 24x - 9$
Step 2: Solve for k using the x^3 terms. $4kx^3 = 20x^3 \rightarrow k = 5$
Step 3: Solve for m using the x^2. Use the $k = 5$. $12x^2 - kmx^2 = -3x^2 \rightarrow 12x^2 - 5mx^2 = -3x^2 \rightarrow$
$-5mx^2 = -15x^2 \rightarrow m = 3$
Step 4: Solve for km. $(3)(5) = 15$
37. **B** To find the solution to a quadratic, set the factors equal to zero and solve for x. We want to do the reverse.
To find the factors, set x equal to the root and solve. $x = -3 \rightarrow x + 3 = 0 \quad x = 6 \rightarrow x - 6 = 0$
$x = 8 \rightarrow x - 8 = 0 \quad (x + 3)$ is the only factor that is an answer choice.
38. **C** Factor and solve for x. $(x^2 - 9)(x + 1)^2 \rightarrow (x + 3)(x - 3)(x + 1)(x + 1) \rightarrow x = -3, 3, -1$
39. **C** Solve for the solutions. $x + a = 0 \rightarrow x = -a \quad x - a = 0 \rightarrow x = a \quad x + b = 0 \rightarrow x = -b$
Three different solutions which are the same as three distinct x-intercepts.
Alternatively, you can pick values for a and b and then solve.
40. **C** The ball will hit the ground at the x-intercept. Solve for the positive x-intercept.
$0 = -x^2 + 10x + 56 \rightarrow 0 = x^2 - 10x - 56 \rightarrow 0 = (x - 14)(x + 4) \rightarrow x = 14, -4$
41. **A** Use the quadratic formula. $x = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{-2 \pm \sqrt{2}}{2}$
42. **D** Step 1: When looking for a maximum or minimum, find the vertex of the parabola. Use $\frac{-b}{2a}$ to solve for the x-coordinate of the vertex. $x = -\frac{30}{-2} = 15$
Step 2: Plug in $x = 15$ to solve for the height. $H(15) = -(15^2) + 30(15) = 225$
43. **B** Find the x-coordinate of the vertex using $x = -\frac{b}{2a}$. Plug in answer choices to find which gives the correct x-coordinate of -1.5. $x = -\frac{3}{2} = -1.5$
Alternatively, pick a point on the graph and plug in the values to solve for b. We will use point $(-3, 3)$.
$3 = -3^2 - 3b + 3 \rightarrow 3 = 9 - 3b + 3 \rightarrow -9 = -3b \rightarrow 3 = b$
44. **D** To solve for x-intercepts, set $y = 0$. B will have an x-intercept of $x = 4.5$. C is factorable and will have x-intercepts of $x = -1, -7$. A is also factorable and will have an x-intercept of $x = 3$. D is not factorable and the discriminant $\sqrt{b^2 - 4ac} < 0$. $\sqrt{16 - 4(1)(6) = \sqrt{-8}}$. Therefore, there is no real solution which means there is no x-intercept.

45. **B** Step 1: Move all of the values to one side to set equal to 0. $5x^2 - 2x - 2 = 0$
Step 2: Use the quadratic formula. $x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(-2)}}{2(5)} = \frac{2 \pm \sqrt{44}}{10} = \frac{2 \pm 2\sqrt{11}}{10} = \frac{1 \pm \sqrt{11}}{5}$
46. **C** Step 1: To find points of intersection, set the equations equal and solve for x.
$-3x + 50 = -x^2 + 6x + 30 \rightarrow x^2 - 9x + 20 = 0 \rightarrow (x-4)(x-5) = 0 \rightarrow x = 4, 5$
Step 2: Correctly interpret there being two possible points of intersection since both values are positive. The baseball and Dave on the zip line will intersect at 4 and 5 seconds.
Alternatively, if you have a graphing calculator, graph the equations to see where they intercept.
47. **A** Step 1: Rewrite first equation in terms of x. $y = x - 18 \rightarrow x = y + 18$
Step 2: Substitute into the second equation and solve for y. $y^2 + (x-14)^2 - 12 = 0 \rightarrow$
$y^2 + (y + 18 - 14)^2 - 12 = 0 \rightarrow y^2 + (y+4)^2 - 12 = 0 \rightarrow y^2 + y^2 + 8y + 16 - 12 = 0 \rightarrow$
$2y^2 + 8y + 4 = 0$
Step 3: Use the quadratic formula to solve for y. $y = \frac{-8 \pm \sqrt{8^2 - 4(2)(4)}}{2(2)} = \frac{-8 \pm \sqrt{32}}{4} = \frac{-8 \pm 4\sqrt{2}}{4} = -2 \pm \sqrt{2}$
48. **C** Step 1: Set equal and then set equal to 0. $6x + 2 = ax^2 + b \rightarrow ax^2 - 6x + b - 2 = 0$
Step 2: Notice that you need to use the discriminant. When the discriminant equals 0, there is one solution. At this point, you can backsolve using the answer choices. The correct answer of C is shown.
$ax^2 - 6x + b - 2 = 0 \rightarrow 9x^2 - 6x + 3 - 2 = 0 \rightarrow 9x^2 - 6x + 1$
Step 3: Use discriminant. $(-6)^2 - 4(9)(1) = 0$ so there is only one solution.